Naïve Coadaptive Cortical Control

Outline

• Naïve coadaptive control: what & why
• Research context
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Naïve coadaptive cortical control

- It is hoped brain-machine interfaces (BMIs) will allow reliable & safe cortical control of prosthetics.
- Past BMI studies used supervised learning, which requires a training signal – something that paraplegics cannot provide!
- Many devices do not have inherent correlates to physical motor control, i.e. wheelchairs: thus need a naive, adaptive algorithm.

{visual, auditory, tactile, µstim} feedback
Research context

- Olds 1965, Fetz 1969 demonstrate that the single unit responses in the motor cortex can be operantly conditioned.
- Shoham et al 2001 demonstrate that SCI patients can modulate activity in M1.
## Research context: Supervised BMIs

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<th>Who/when</th>
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</thead>
<tbody>
<tr>
<td>Chapin 1999</td>
<td>Nature Neuroscience v.2 no. 7 664-670</td>
<td>rat</td>
<td>PCA-&gt;ANN, 20ms bin</td>
<td>&lt;1</td>
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<td>Wessberg 2000</td>
<td>Nature v 208 361-365</td>
<td>owl monkey</td>
<td>Wiener, ANN, 100ms bin</td>
<td>&lt;1, 3</td>
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<td>Taylor 2002</td>
<td>Science V 296 1829-1832</td>
<td>2 rhesus macaques</td>
<td>coadaptive, 90ms normalized bin, adhoc/gradient descent</td>
<td>&lt;3, 3 bits 64 recorded, 39-17 used</td>
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<td>Serruya 2002</td>
<td>Nature 416 141-142</td>
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<td>Carmina 2003</td>
<td>PLoS Biology V 1(2) 193-208</td>
<td>2 rhesus macaques</td>
<td>wiener, 100ms bin, 10 lag, block train</td>
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<td>Paninski 2003</td>
<td>J Neurophysiology 91 515-532</td>
<td>3 rhesus macaques</td>
<td>Bayes, conditional probabilities modeled w gaussains, wiener prediction</td>
<td>2</td>
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<td>Musallam 2004</td>
<td>Science 305 258-262</td>
<td>3 monkeys</td>
<td>Harr wavelet decomposition-&gt;Bayes rule via histogram data base - adaptive</td>
<td>2-3 bits 8-16</td>
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<tr>
<td>Olson 2005</td>
<td>IEEE Trns. Neural Sys. Rehabilitation 13(1) 72-80</td>
<td>4 rats</td>
<td>block-update SVM</td>
<td>1 bit</td>
<td>8-10</td>
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</table>
Wiener filter

• In general, each study used an implementation of an adaptive filter to map neuronal firing patterns to cursor/prosthetic control.

• The simplest assumption is that the firing rate is linearly related to \{position, velocity, force\}:

\[
\hat{y}(t) = \hat{y}_{dc} + \sum_{i=0}^{I} \sum_{n} h_n(i) \hat{x}_n(t - i) + \hat{e}(t)
\]


\begin{itemize}
  \item \text{position/velocity/force}
  \item \text{dc term}
  \item \text{weights}
  \item \text{error}
  \item \text{binned neuronal firing}
\end{itemize}

or:

\[
Y = HX
\]

Wiener solution:

\[
H = inv(X^T X)(X^T)Y
\]

\begin{itemize}
  \item \text{autocorrelation}
  \item \text{crosscorrelation}
\end{itemize}

The wiener filter is block-update, but the same optimal linear solution can be found iteratively by LMS (least mean squares) or RLS (recursive least squares)
Limitations of Wiener/ optimal linear filters

- While you can predict position, velocity, and force independently, you cannot predict them in a self-consistent manner.
- Solution: give the ‘plant’ memory: dependence on past states (wiener = linear dependence on past/present neural firing, state memory implicit in lagged input)

\[ \hat{x}_{i+1} = A\hat{x}_i + Gu_i \]
\[ \hat{y}_i = H\hat{x}_i \]

\[ \hat{x}_{i+1} = \begin{bmatrix} y_{i+1} \\ y_i \\ y_{i-1} \\ \vdots \\ y_{i-n+2} \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & \cdots & a_{n-2} & a_{n-1} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} y_i \\ y_{i-1} \\ y_{i-2} \\ \vdots \\ y_{i-n+1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_i \]
Kalman filter

• The Kalman filter is the optimal estimator when the process is (or is well modeled by) linear state and measurement update and the process/measurement noise is stationary and Gaussian.

• However, even in suboptimal conditions (i.e. most of the time) the Kalman filter is straightforward and works pretty well – hence it is a favored tool.

• When used in a BMI, the state \( x \) is that of the cursor/prosthetic, and the measurement \( y \) are the recorded neural signals.

http://www.cs.unc.edu/~welch/kalman/kalmanBiblio.html
Kalman filter

- As per the model, the filter is two-step: an *a priori* state estimate given knowledge of the process, and an *a posteriori* state estimate given the measurement.

\[
\hat{x}_k = \hat{x}_k^- + K (z_k - H \hat{x}_k^-)
\]

- The Kalman filter was derived by minimizing the *a posteriori* error covariance.

\[
ea_p: \quad e_k^- = x_k - \hat{x}_k^- \\
\quad P_k^- = E[e_k^- e_k^-^T]
\]

\[
ea_p: \quad e_k = x_k - \hat{x}_k \\
\quad P_k = E[e_k e_k^T]
\]
Kalman filter

\[\begin{align*}
e_k &= x_k - \hat{x}_k \\
\hat{x}_k &= \hat{x}_k^- + K(z_k - H\hat{x}_k^-) \\
e_k &= x_k - \hat{x}_k^- + K(z_k - H\hat{x}_k^-) \\
z_k &= Hx_k + v_k \\
p(v) &\sim N(0, R); \ E[v_kv_k^T] = R \\
P_k^- &= E[e_k^-e_k^{-T}] \\
P_k &= E[e_ke_k^T] \\
\frac{\partial P_k}{\partial K} &= 0 \\
K_k &= P_k^-H^T(HP_k^-H^T + R)^{-1}
\end{align*}\]
Kalman filter

- The filter is recursive, but four matrices must be estimated beforehand:
  - $A$ (process update)
  - $H$ (measurement)
  - $Q$ (process noise covariance)
  - $R$ (measurement noise covariance)

**Time Update ("Predict")**

1. Project the state ahead
   $$\hat{x}_k^- = A\hat{x}_{k-1}$$
2. Project the error covariance ahead
   $$P_k^- = AP_{k-1}A^T + Q$$

**Measurement Update ("Correct")**

1. Compute the Kalman gain
   $$K_k = P_k^- H^T (HP_k^H H^T + R)^{-1}$$
2. Update estimate with measurement $z_k$
   $$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H\hat{x}_k^-)$$
3. Update the error covariance
   $$P_k = (I - K_k H)P_k^-$$

Initial estimates for $\hat{x}_{k-1}$ and $P_{k-1}$
The subjects

* 16 recording site Si 

µ electrodes each in Primary motor cortex, forelimb area (mean AP:2.5, mean ML:2.4 ; auditory cortex: AP:-4 ML:7, units in mm)

Chronic Neural Recording Using Silicon-Substrate Microelectrode Arrays Implanted in Cerebral Cortex.
Their task

5-15 seconds, to make sure the modulation is a response & not periodic

window is 17% of the logarithmic workspace – allows naïve users to acquire target 15-20% of trials by chance.

Kalman filter is updated between trials

extracted audio cursor position (idealized, actually piecewise constant @ sampling period of 90ms)

Baseline Criterion

Baseline (1.26s)

Cue (0.9s)

Response (variable, up to 4.5s)

Response Criterion

Inter-trial (Random)

Sliding Hold

Their task

Output Frequency

4 kHz

6 kHz

8 kHz

10 kHz

(log scale)
Discrimination task

2 of the 6 rats had to discriminate between a 1.5kHz and 10kHz tone.

“auditory analog of a center-out reaching task” ala Georgopoulos
Coadaptive algorithm

• state model
  \[ x_{tk} = Ax_{t(k-1)} + w_{t(k-1)} \]
  \[ w_{tk} \sim N(0, W_t) \]
  \( x_{tk} \) is a scalar, \( t \) indexes trial, \( k \) indexes the 90ms bin within a trial.

• measurement model:
  \[ z_{tk} = H_t x_{tk} + q_{tk} \]
  \[ q_{tk} \sim N(0, Q_t) \]

• Thus, three matrices must be adapted online: \( H, Q \) and \( W \).
Block-update

concatenate the past 10 trials to form an observation block

\[
l = \arg \max_j \left( \max_{c \in \{1 \ldots C\}} \text{corr}(X_j, Z) \right)
\]

\[
\text{corr}(x, z) = \frac{\sum_i (x_i - \bar{x})(z_i - \bar{z})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (z_i - \bar{z})^2}}, \quad \bar{x} = \frac{1}{n} \sum_{i=0}^{n} x_i
\]
Block matrix update

• Measurement transform matrix is found via the Wiener solution: \( \hat{H}_{(t+1)} = (ZX^T)(XX^T)^{-1} \)

• Process and measurement noise covariance matrices are estimated from the actual signals:

\[
\hat{W}_{(t+1)} = \frac{(X_{2:N} - AX_{1:N-1})(X_{2:N} - AX_{1:N-1})}{(N - 1)}
\]

\[
\bar{Q}_{(t+1)} = \frac{(Z - \bar{H}_{(t+1)}X)(Z - \bar{H}_{(t+1)}X)^T}{N}
\]

• Initial matrices are randomized.

there was a typo in the manuscript which I had to correct.
Overall schematic

\[ x_{tk} = Ax_{(k-1)} + w_{tk} \]
\[ z_{tk} = H_t x_{tk} + q_{tk} \]

Kalman Filter

\[ \hat{x}_{tk}^- = A\hat{x}_{(k-1)} \]
\[ P_{tk}^- = A P_{(k-1)} A^T + W_t \]
\[ K_{tk} = P_{tk}^- H_t^T (H_t P_{tk}^- H_t^T + Q_t)^{-1} \]
\[ \hat{x}_{tk} = \hat{x}_{tk}^- + K_{tk} (z_{tk} - H_t \hat{x}_{tk}^-) \]
\[ P_{tk} = (I - K_{tk} H_t) P_{tk}^- \]

Online Adaptation

\[ [H_{(t+1)}, W_{(t+1)}, Q_{(t+1)}] = f(X, Z) \]
Results!

- First, the simpler 10kHz target task – all 6 rats succeeded at this, as quantified by a stimulus randomization test.

Stimulus randomization test:
1. discard all late trials.
2. shuffle the binned neural data.
3. Run the coadaptive algorithm on the shuffled data & count how many times target was attained.
4. Repeat 2 & 3 to build up a distribution of correct with shuffled rat data.
5. Fit with a Gaussian.
mean number of units recorded: 11.5. Units resorted at the beginning of every session.
Evidence supporting ⋄ of control

1. Cursor histograms:

(a) Session 1 (Naïve)
(b) Session 7 (Early Learning)
(c) Session 24 (Late Learning)

2. Psychometric curve:

warning: one rat!
Evidence supporting evidence of learning

warning: one (different) rat!
Two target bilateral task

- Two rats were trained on the task, but only one rat mastered it.
- Offline classification using a support vector machine (SVM) showed that rat # 10 could distinguish the targets, but was unable to control the cursor.
Two target bilateral task

- Rat #9 was able to modulate recorded cells bidirectionally to acquire both targets. An alternative strategy would be to have one set of neurons with positive weight move the target toward 10kHz, and another with negative weight to move toward 1.5kHz.

Line thickness indicates standard error. Mean trajectory plots.
Conclusions

• A naïve user can learn to control a one-dimensional cursor using single and multiunit activity in the cortex.
• A naïve, coadaptive model is capable of extracting relevant firing modulations, even when the recorded units change between days. Goal-directed behavior is needed, but a training signal as in supervised learning is not.
• Their model can be extended to include a nonlinear measurement function and the possibility of including new units during free movement via the methods outlined in Eden et. al. 2004.
References


my thanks to the Duke neuroengineering journal club for commentary of this paper.